High-Quality Extraction of Isosurfaces from Structured and Unstructured Grids

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Overview

- Simple, fast method for generating good triangular approximations to isosurfaces
Motivation

- Processing pipeline

- Marching Cubes
  - Simplification
  - Smoothing
Motivation

- Processing pipeline

- Marching Cubes
- Simplification
- Smoothing
Motivation

- Processing pipeline

Marching Cubes → Simplification → Smoothing
Motivation

- Processing pipeline

AFront
Overview

- Arbitrary topology
- Works with raw data
Meshes are important
Meshes are important
Related Work

- Marching Cubes and variants
  - Cline and Lorensen
  - Efficiency: Wilhems et al, Cignoni et al, Shen et al
  - Correctness: Nielson, Lewiner et al

- Force-based systems
  - Particles: Crossno et al, Meyer et al

- Hybrid systems
  - Wood et al, Guskov et al
  - Gavriliu et al.

- Many, many others, see paper!
How do we size the elements?
How do we size the triangles?
Guidance Field

Schreiner et al., Eurographics 2006
Guidance Field for Isosurface Extraction

- We use the spatial filter design formulation of Kindlmann et al.
- Geometry tensor
  - Compute curvature from gradient, Hessian

\[
P = I - nn^T
\]

\[
H = \begin{bmatrix}
\frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial x \partial z} \\
\frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} & \frac{\partial^2 f}{\partial y \partial z} \\
\frac{\partial^2 f}{\partial x \partial z} & \frac{\partial^2 f}{\partial y \partial z} & \frac{\partial^2 f}{\partial z^2}
\end{bmatrix}
\]

\[
G = PHP/|\nabla f|
\]

\[
\kappa_{1,2} = \frac{T \pm \sqrt{2F^2 - T^2}}{2}
\]

T trace of G
F Frobenius norm of G
Guidance Field for Isosurface Extraction

- Determine if set of samples is dense enough
  - We want to bound minimum triangle size
  - Bound max curvature (spectral radius of geometry tensor)
    - Upper bound on *any* consistent matrix norm of Hessian
    - Lower bound on gradient magnitude

\[
\begin{align*}
  r(M) &\leq \|M\| \\
r(G) &= \kappa_{\text{max}} \\
&\leq \|G\| \\
&\leq \|\mathbf{PHP}/\nabla f\| \\
r(G) &\leq \frac{2\sqrt{3}}{\|\nabla f\|} \|H\| \quad \text{(for Frobenius matrix norm)}
\end{align*}
\]
Culling the Guidance Field

These are immaterial!

These are relevant
Culling the Guidance Field
Culling the Guidance Field
Culling the Guidance Field
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Culling the Guidance Field
Culling the Guidance Field
Culling the Guidance Field

Real version features
4D cones
Culling the Guidance Field
Since \(|n| = 72\) and so \(nx_0 + \ldots\) the same interpolation could be plugged into our algorithm. We have been careful in our implementation to make this an
Results

B-Spline
Marching Tetrahedra
Results

Nielson Interpolation
Results

Moving Least Squares
Discussion

- Method is appropriate if subsequent processing is necessary
- One pass algorithm produces results comparable to global methods
- Output mesh is dependent on the isosurface itself, and not the domain on which it is defined

- Requires the gradient of function to be defined
  - True for all manifold isosurfaces
  - No sharp features
Future Work

- Out of core meshing of gigantic data sets
  - Particularly for regular grids
  - Output already streamed, stream input
  - Control interaction between global guidance field and input stream

- Bound quality of all triangles
  - Not just those that create new vertices
Acknowledgments

- NSF grants
  - CCF-0401498
  - EIA-0323604
  - OISE-0405402
  - IIS-0513692
  - CCF-0528201
- DOE
- Sandia National Laboratories
- Lawrence Livermore National Laboratory
- IBM Faculty Award
- University of Utah Seed Grant

http://www.volvis.org for volumes
Thank you!

Questions?
## Results

<table>
<thead>
<tr>
<th>Model</th>
<th>Alg.</th>
<th>(\rho)</th>
<th>(\eta)</th>
<th>time</th>
<th># tris</th>
<th>Histogram</th>
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## Results

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