Tutorial: Feature Oriented Methods in Flow Visualization

Topological Methods

Xavier Tricoche
Scientific Computing and Imaging Institute
University of Utah

IEEE Visualization 2004
Austin, TX

Outline

- Motivation
- Theory basics
- Implementation
- Applications
- Pros and cons

Motivation

- Synthetic depiction of complex flow structures

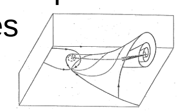


Fig. (38) Singular streamsurface, representing a vortex that leaves the surface of a body. No critical points in the plane of symmetry.

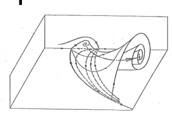


Fig. (39) Singular streamsurface representing a vortex that causes separation in the plane of symmetry.

- Efficient computation of global features
- Rich mathematical framework
- Versatile tool

Dallmann, ***Topological Structures of Three-Dimensional Flow Separation***. DFVLR-AVA Bericht Nr. 221-82 A 07,April 1983.

Outline

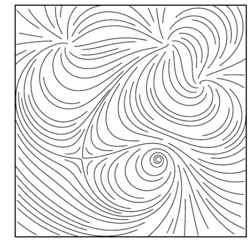
- Motivation
- **Theory basics**
- Implementation
- Applications
- Pro and Contra

Dynamical Systems

- For a Lipschitz continuous vector field
  $$\vec{f} : (I \subseteq \mathbb{R}) \times (U \subseteq \mathbb{R}^n) \to \mathbb{R}^n$$
- Differential equation (ODE)
  $$\frac{d\vec{x}}{dt} = \vec{f}(t, \vec{x}) \qquad \text{steady}$$
- Flow
  $$\begin{cases} \frac{d}{dt}\vec{\phi}(\vec{x}, t)\Big|_{t=\tau} = \vec{f}(\tau, \vec{\phi}(\vec{x}, \tau)) \\ \vec{\phi}(0, \vec{x}) = \vec{x} \quad \leftarrow \text{initial condition} \end{cases}$$

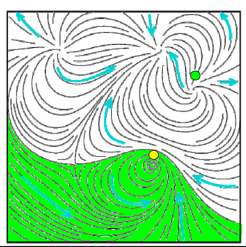
Phase Portrait

- Integral curve
  $$\vec{x}(\tau) = \vec{\phi}(\tau, \vec{x}_0) = \vec{x}_0 + \int_0^\tau \vec{f}(t, \vec{x}(t))dt$$
- Streamline / pathline
- **Phase portrait**: set of all integral curves



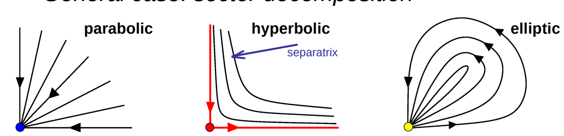
Limit Sets and Basins

- **Topology**: structure of phase portrait w.r.t. asymptotic behavior of integral curves
- **Limit set**: locus of convergence $t \to \pm\infty$ al curves when
- α / ω -**basin** of a limit set (outsets/insets)
- **Separatrices**

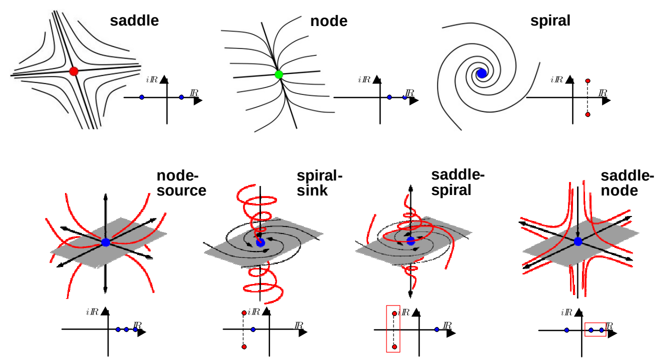


Critical Points

- Equilibrium $\vec{f}(P_0) = \vec{0}$
- Local flow structure:
  - *Linear case: determined by eigenvalues of Jacobian*
    - Sign of real part: + ⇔ , - ⇔
    - Complex part:
  - *General case: sector decomposition*

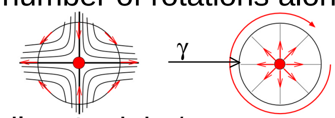


Critical Points



Poincaré Index

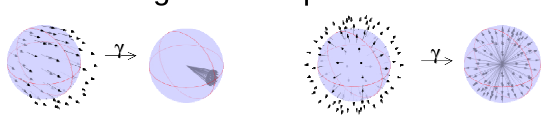
- Gauss map: $\gamma: \mathbb{R}^n \to S^{n-1},\ \gamma(\vec{v}) = \frac{\vec{v}}{||\vec{v}||},\ \vec{v} \neq \vec{0}$
- Index of closed region with boundary Γ: # of coverings of S^{n-1} around $\vec{0}$ by $\gamma \circ \vec{f}(\Gamma)$
- Additive - index = 0 for homogeneous flow
- 2D: number of rotations along curve



- Saddle: -1, sinks/sources: +1

Poincaré Index

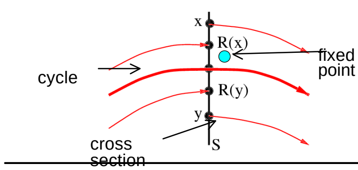
- Gauss map: $\gamma: \mathbb{R}^n \to S^{n-1},\ \gamma(\vec{v}) = \frac{\vec{v}}{||\vec{v}||},\ \vec{v} \neq \vec{0}$
- Index of closed region with boundary Γ: # of coverings of S^{n-1} around $\vec{0}$ by $\gamma \circ \vec{f}(\Gamma)$
- Additive - index = 0 for homogeneous flow
- 3D: coverings of unit sphere



Linear case: index = sign(det J): source +1, sink -1, saddle ±1

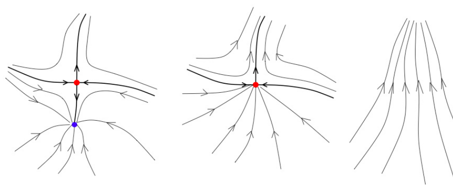
Cycles and Tori

- Periodic solution: closed curve or torus
- Sink / source
- Separatrix
- Poincaré map (1D/2D)



Bifurcations

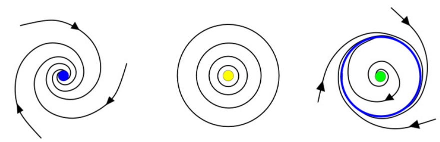
- Structural changes: nature and position of critical points, connectivity
- Local: index preserving



Pairwise Cancellation
Saddle + Sink/Source
Index: +1 / -1 ® 0

Bifurcations

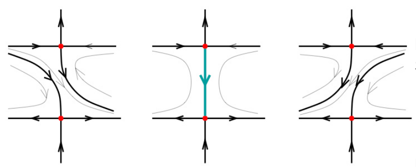
- Structural changes: nature and position of critical points, connectivity
- Local: index preserving



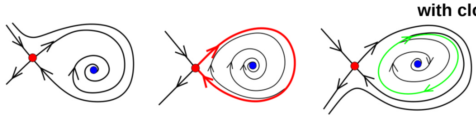
Hopf bifurcation:
sink ® source+cycle
cycle is sink
Index: +1 ® +1

Bifurcations

- Global: transforms overall connectivity



Basin bifurcation:
Saddle-saddle connection



Homoclinic bifurcation
with closed orbit

Outline

- Motivation
- Theory basics
- **Implementation**
- Applications
- Pro and Contra

Topology Extraction - *Steady Case*

- Cell-wise critical point extraction
  - *Linear, bilinear, trilinear interpolation*
- Type characterization
  - *Compute Jacobian (cell-wise defined)*
  - *Compute eigenvalues and eigenvectors*
- Integrate separatrices
  - *Streamlines*
  - *Stream surfaces*

Topology Extraction - *Steady Case*

- Helman, Hesselink, ***Representation and Display of Vector Field Topology in Fluid Flow Data Sets***. Computer, Vol. 22, No. 8, 1989: 27-36.
- Helman, Hesselink, ***Visualizing Vector Field Topology in Fluid Flows***. IEEE Computer Graphics and Applications, Vol. 11, No. 3, 1991: 36-46.
- Globus, Levit, Lasinski, ***A Tool for Visualizing the Topology of Three-Dimensional Vector Fields***. IEEE Visualization '91 Proceedings: 33-40.

Feature Oriented Methods in Flow Visualization: **Topological Methods**

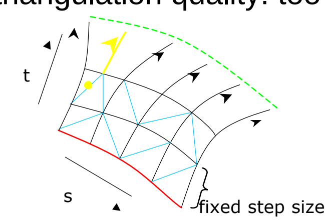
Streamline Integration

- High accuracy required close to critical points: strong variations of norm and direction
- Tetrahedral grids: piecewise analytical formula
  Nielson, Jung, ***Tools for Computing Tangent Curves for Linearly Varying Vector Fields over Tetrahedral Domains***, TVCG 5(4):360-372, 1999
- General case: Runge Kutta 4th order with adaptive step size
  Press, Flannery, Teukolsky, Vetterling, ***Numerical Recipes in C (2nd ed.)***, Cambridge University Press, 1992
- Remark: smoothing effect of RK-4, nice for CFD applications (noise)

Feature Oriented Methods in Flow Visualization: **Topological Methods**

Stream Surface Integration

- Hultquist: uses RK2 with fixed step size to control triangulation quality: too inaccurate



Hultquist, ***Constructing Stream Surfaces in Steady 3D Vector Fields***, IEEE Visualization '92 Proceedings,171-178

Feature Oriented Methods in Flow Visualization: **Topological Methods**

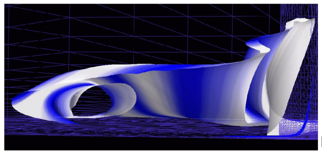
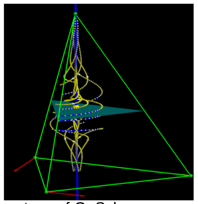
Stream Surface Integration

- Implicit Stream Surfaces:
  - *Rely on seeding strategy*
  - *Accuracy depends on grid resolution*

- van Wijk, ***Implicit Stream Surfaces***, IEEE Visualization '93 Proceedings, 1993, 245-252.
- Mahrous, Bennett, Scheuermann, Hamann, Joy, ***Topological Segmentation in Three-Dimensional Vector Fields***, IEEE TVCG 10(2), 2004, 198-205.

Stream Surface Integration

- Extension to stream surfaces of Nielson's technique for piecewise analytic streamline integration

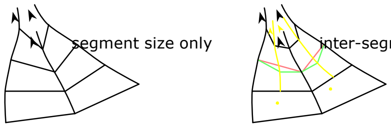



Pictures courtesy of G. Scheuermann

- Scheuermann, Bobach, Hagen, Mahrous, Hamann, Joy, Kollmann, ***A Tetrahedra-Based Stream Surface Algorithm***. IEEE Visualization '01 Proceedings, 151-158.
- Nielson, Jung, ***Tools for Computing Tangent Curves for Linearly Varying Vector Fields over Tetrahedral Domains***, TVCG 5(4):360-372, 1999

Stream Surface Integration

- Improved Hultquist's method:
  - *RK4 / adaptive step size + arc length control*
  - *Control of front curvature: avoid crease*



Garth, Tricoche, Salzbrunn, Bobach, Scheuermann, ***Surface Techniques for Vortex Visualization***, Joint Eurographics - IEEE TCVG Symposium on Visualization, 2004

Saddle Connectors

- To avoid occlusion: replace separating stream surfaces by streamlines of interest
- Saddle connector: streamline connecting two saddle points of opposite indices (inflow vs. outflow plane)
- Found by computing intersection of polygonal fronts integrated from both saddle points

Theisel, Weinkauf, Hege, Seidel, ***Saddle Connectors: An Approach to Visualizing the Topological Skeleton of Complex 3D Vector Fields***, IEEE Visualization 2003 Proceedings, 225-232.

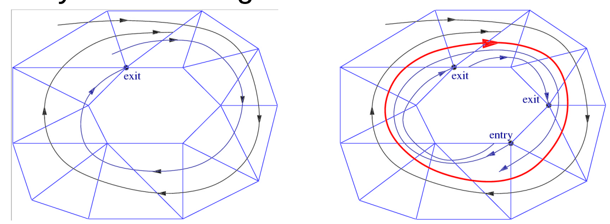
Saddle Connectors

Pictures courtesy of H. Theisel
http://www/mpi-sb.mpg.de/~theisel/gallery/

Theisel, Weinkauf, Hege, Seidel, ***Saddle Connectors: An Approach to Visualizing the Topological Skeleton of Complex 3D Vector Fields,*** IEEE Visualization 2003 Proceedings, 225-232.

Cycle Extraction

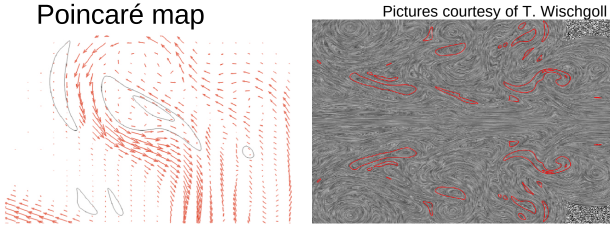
- Poincaré-Bendixson theorem
- Check along edges if flow is *captured* by cell-wise region



Cycle Extraction

- Cycle finally extracted as fixed point of Poincaré map

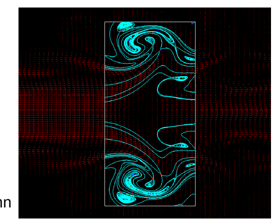
Pictures courtesy of T. Wischgoll



- Wischgoll, Scheuermann, ***Detection and Visualization of Closed Streamlines in Planar Flows***, TVCG 7(2):165-172
- Wischgoll, Scheuermann, ***Locating Closed Streamlines in 3D Vector Fields***, Joint Eurographics - IEEE TCVG Symposium on Data Visualization 2002: 227-232.

Local Topology

- Inflow / outflow regions on boundary are limit sets of local topology
- Tangential flow: additional separatrices



Picture courtesy of G. Scheuermann

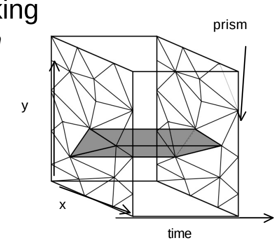
- Scheuermann, Hamann, Joy, Kollmann, ***Visualizing Local Vector Field Topology***, Journal of Electronic Imaging 9(4):109-116, 2000
- Weinkauf, Theisel, Hege, Seidel, ***Boundary Switch Connectors for Topological Visualization of Complex 3D Vector Fields.*** Joint Eurographics and IEEE TCVG Symposium on Visualization (2004)

Topology Tracking

- Visualize continuous evolution of parameter-dependent topology
- Requires time interpolation: (n+1)D grid
- Detect, identify and display bifurcations
- Methods:
  - *Cell-wise singularity tracking*
  - *Feature flow fields*

Topology Tracking

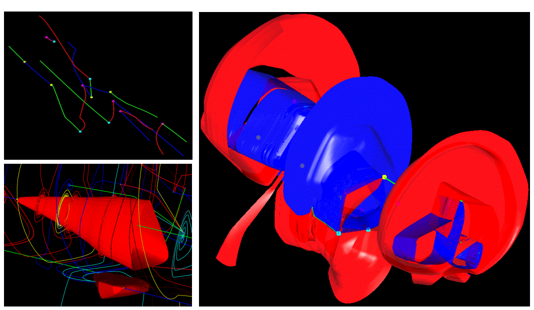
- Cell-wise singularity tracking
  - *n+1D linear interpolation*
  - *Compute paths*
  - *Creations/cancellations occur on faces*
  - *Integrate separatrices*
  - *Monitor connectivity*



- Tricoche, Wischgoll, Scheuermann, Hagen, ***Topology Tracking for the Visualization of Time-Dependent Two-Dimensional Flows.*** In Computer & Graphics (26), Elsevier, 2002.
- Garth, Tricoche, Scheuermann, ***Tracking of Vector Field Singularities in 3D Unstructured Data Sets***. IEEE Visualization '04 Proceedings.

Topology Tracking

- Cell-wise singularity tracking



Topology Tracking

- Feature flow fields:
  - *Track path of critical points through streamline integration in a vector field defined over the (n+1)D space-time domain*

$$\vec{f}(x,y,t) = \begin{pmatrix} u(x,y,t) \\ v(x,y,t) \end{pmatrix} \qquad \vec{g} = \nabla u \times \nabla v$$

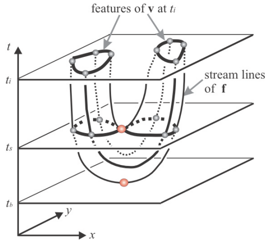
  - *The value of $\vec{f}$ (e.g. $\vec{0}$) is constant along streamlines of $\vec{g}$*

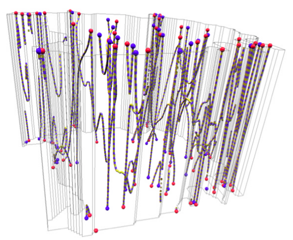
- Theisel, Seidel, ***Feature Flow Field***. Proceedings of Joint Eurographics and IEEE TCVG Symposium on Visualization (2003)

Topology Tracking

- Feature flow fields:

Pictures courtesy of H. Theisel
http://www/mpi-sb.mpg.de/~theisel/gallery/





Theisel, Seidel, ***Feature Flow Field***. Proceedings of Joint Eurographics and IEEE TCVG Symposium on Visualization (2003)

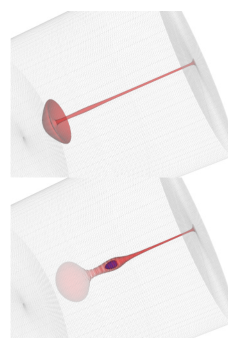
Outline

- Motivation
- Theory basics
- Implementation
- **Applications**
- Pro and Contra

Vortex Breakdown Analysis

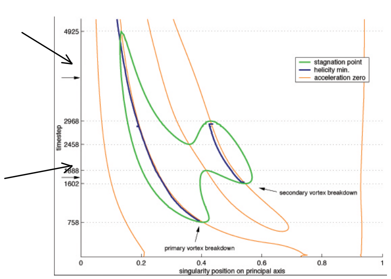
- Efficient analysis of large transient data
- Identification of interesting time steps
- Investigation of interplay between different physical quantities
- Visualization of essential structures
- **Track critical points of velocity with extrema of acceleration and helicity**
- **Handle noisy and turbulent data**

Vortex Breakdown Analysis



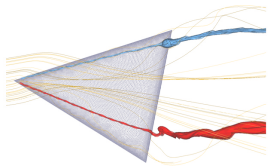
Vortex breakdown phenomenon in a closed cylindrical container

Data set courtesy of M. Ruetten, DLR Göttingen



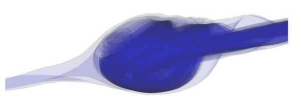
Garth, Tricoche, Scheuermann, ***Tracking of Vector Field Singularities in 3D Unstructured Data Sets***. IEEE Visualization '04 Proceedings.

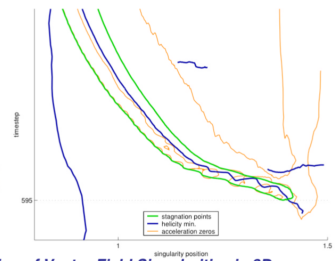
Vortex Breakdown Analysis



Vortex breakdown phenomenon in a delta wing simulation (right bubble)

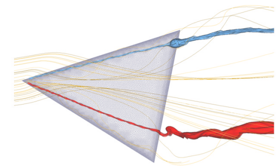
Data set courtesy of M. Ruetten, DLR Göttingen





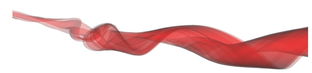
Garth, Tricoche, Scheuermann, ***Tracking of Vector Field Singularities in 3D Unstructured Data Sets***, IEEE Visualization '04 Proceedings.

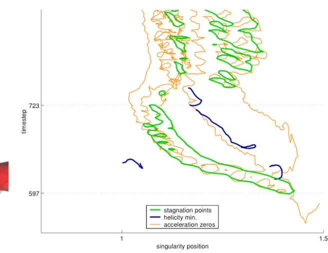
Vortex Breakdown Analysis



Vortex breakdown phenomenon in a delta wing simulation (right bubble)

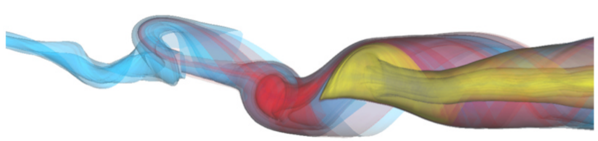
Data set courtesy of M. Ruetten, DLR Göttingen

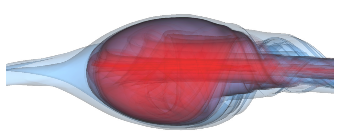




Garth, Tricoche, Scheuermann, ***Tracking of Vector Field Singularities in 3D Unstructured Data Sets***, IEEE Visualization '04 Proceedings.

Vortex Breakdown Analysis





Vortex breakdown phenomenon in a delta wing simulation Separation surfaces in left and right bubbles

Data set courtesy of M. Ruetten, DLR Göttingen

Garth, Tricoche, Scheuermann, ***Tracking of Vector Field Singularities in 3D Unstructured Data Sets***, IEEE Visualization '04 Proceedings.

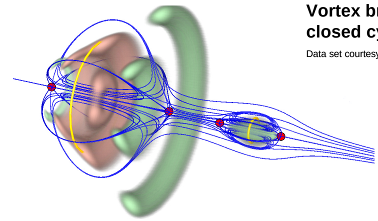
Volume Exploration

- Vortical flows exhibit intricate structures
  - *Multiple vortices*
  - *Recirculation bubbles*
- 3D topology is typically cluttered
- Stream surfaces are insufficient
- **Idea: unfold complex structures**

Volume Exploration

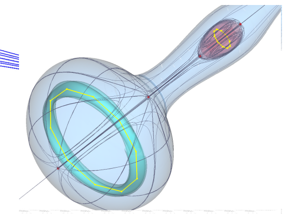
- Method: walk through 3D structures
- Moving cutting plane
  - *smoothly sweeping along prescribed curve*
  - *Automatic orientation (maximize crossing flow)*
- Track 2D topology of projected flow
  - *Curve = parameter space*
  - *Filter out small-scale (“time”, space) features*
- Map geometry back in 3D physical space
- Combined visualization with DVR for context

Volume Exploration



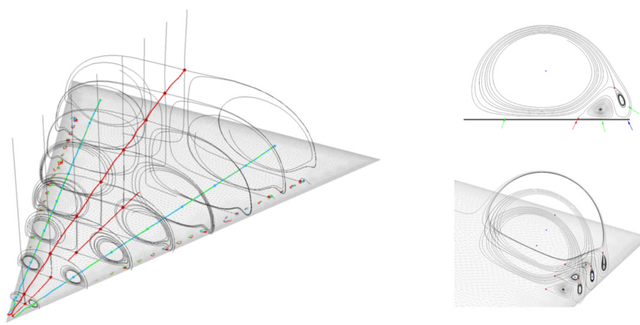
Vortex breakdown phenomenon in a closed cylindrical container

Data set courtesy of M. Ruetten, DLR Göttingen



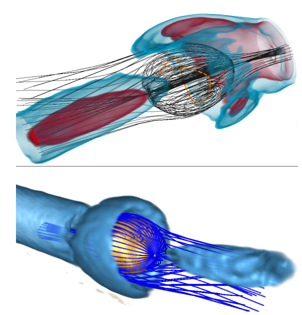
Tricoche, Garth, Kindlmann, Deines, Scheuermann, Rütten, Hansen, ***Visualization of Intricate Flow Structures for Vortex Breakdown Analysis***. IEEE Visualization ‘04 Proceedings.

Volume Exploration



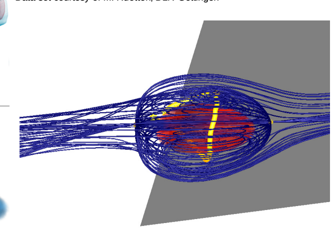
Tricoche, Garth, Kindlmann, Deines, Scheuermann, Rütten, Hansen, ***Visualization of Intricate Flow Structures for Vortex Breakdown Analysis***. IEEE Visualization ‘04 Proceedings.

Volume Exploration



Vortex breakdown phenomenon in a delta wing simulation: right bubble

Data set courtesy of M. Ruetten, DLR Göttingen



Tricoche, Garth, Kindlmann, Deines, Scheuermann, Rütten, Hansen, ***Visualization of Intricate Flow Structures for Vortex Breakdown Analysis***. IEEE Visualization ‘04 Proceedings.

Outline

- Motivation
- Theory basics
- Implementation
- Applications
- **Pros and cons**

Topology is a Powerful Approach

- Very efficient computation
- Abstract representation:
  - *Synthetic depiction*
  - *Data reduction for interpretation*
  - *Unified framework: scalars, vectors, tensors, multi-fields*
- Building block for specific visualization techniques

But Topology Alone is Not Sufficient

- Topological structures ≠ *"features"*
- Needs right coordinate frame
- Can be non-intuitive
- Clutter and occlusion problems in 3D

Acknowledgments

- Christoph Garth
  University of Kaiserslautern
- Gerik Scheuermann
  University of Leipzig
- Holger Theisel
  Max-Planck-Institut Saarbrücken
- Thomas Wischgoll
  UC Irvine